

BULETINUL INSTITUTULUI POLITEHNIC DIN IAȘI
Publicat de
Universitatea Tehnică „Gheorghe Asachi” din Iași
Tomul LIX (LXIII), Fasc. 2, 2013
Secția
CONSTRUCȚII DE MAȘINI

MAXIMUM PERFORMANCES OF THE STIRLING MACHINE WORKING AS MOTOR-DRIVEN COMPRESSOR

BY

VLAD MARIO HOMUTESCU*

“Gheorghe Asachi” Technical University of Iași,
Department of Mechanical Engineering and Automotive

Received: May 9, 2012

Accepted for publication: September 3, 2013

Abstract. The functioning of an engine-driven Stirling compressor is analyzed using two isothermal physico-mathematical models. The maximum performances of a beta-type machine and the performances that can be achieved by the theoretical engine-driven Stirling compressor (working after the Stirling thermodynamic cycle) are obtained. These maximum performances can be used as references for comparing different physical embodiments of engine-driven Stirling compressors.

Key words: Stirling machine, engine-driven compressor, maximum performances, isothermal models.

1. Introduction

The engine-driven Stirling compressor (EDSC) is a Stirling machine that rises the pressure of a gas and, simultaneously, yields work to the engine crankshaft. The EDSC is constructed after the general scheme of the Stirling engine, and is equipped with suction (or intake) and discharge valves and lines. The working agent must be the gas intended to be compressed.

The valves are opened and closed automatically by the pressure difference acting on the valve discs, in the same way as at the regular reciprocating

*Corresponding author; *e-mail*: mariohomutescu@gmail.com

compressor. The valves must be placed near the heat exchanger serving as cooler. This placement eliminates the need of a final cooler for the compressed gas.

2. Physico-Mathematical Model of the Beta-Type Engine-Driven Stirling Compressor

The scheme of a beta-type EDSC with rhombic drive is presented in Fig. 1. The main components of the Stirling engine are easily recognizable (Homutescu *et al.*, 2003): cylinder 15 with the power piston 10 and with displacer 4, heat exchangers 2 - heater, 5 - regenerator and 6 - cooler. Furthermore, the EDSC is equipped with the suction valve (SV) 14 and discharge valve (DV) 7 and with the corresponding suction and discharge lines.

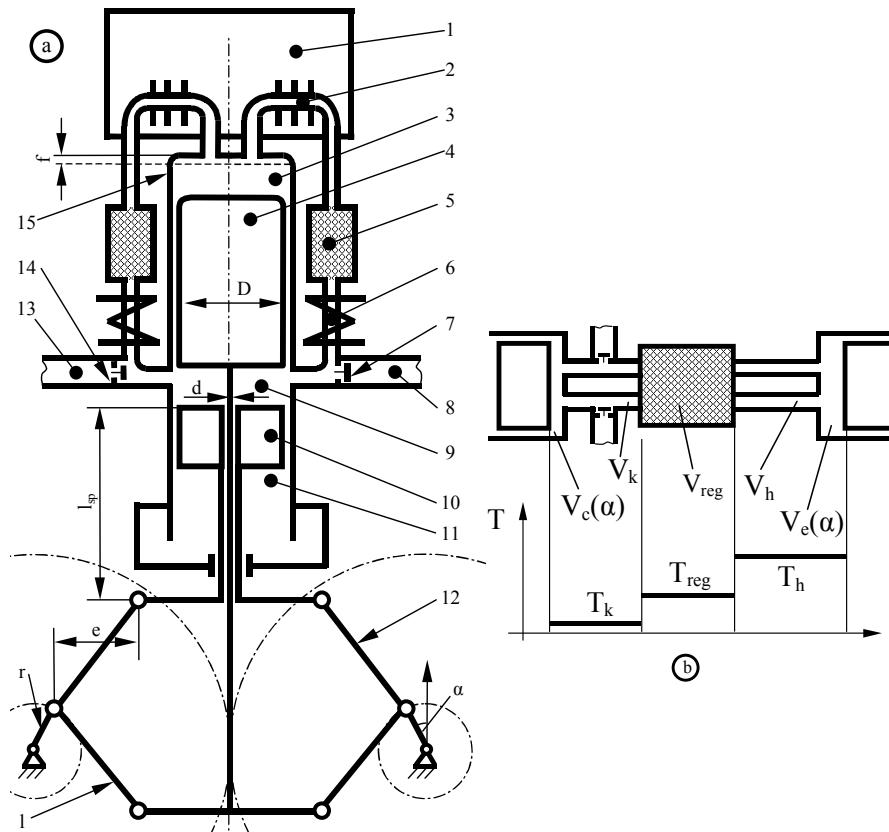


Fig. 1 – Stirling engine-driven compressor (a) and the isothermal model (b):
 1 - combustion chamber; 2 - heater; 3 - expansion chamber; 4 - displacer;
 5 - regenerator; 6 - cooler; 7 - discharge valve; 8 - discharge line; 9 - compression
 chamber; 10 - power piston; 11 - buffer space; 12 - drive mechanism; 13 - suction valve;
 14 - suction line; 15 - cylinder; α - crankshaft angle.

The isothermal mathematical model for thermal calculation (Homutescu *et al.*, 2001) is based on the conditions referred to as G. Schmidt hypotheses (Popescu, 2001): the working fluid is the ideal gas, all processes occur ideally, the temperature inside the expansion space and the heater is constant and equal to the one of the displacer and the cylinder around its warm zone, the temperature inside the compression space and the cooler is constant and equal to the one of the power piston and the cylinder around its cold zone, the heat regenerator temperature is constant and equal to the two spaces temperature logarithmic mean, the momentary pressure is the same inside all engine chambers and the pistons move accordingly to sinusoidal laws. The hypotheses concerning the temperatures inside the engine are shown as well in graphical manner on Fig. 1 *b*. In the paper the fore mentioned hypotheses are followed except for considering the real movement laws for the two pistons, laws differing of course from the sinusoidal ones. On Fig. 1 *b* the letters V , T were used for marking the volumes and temperatures and the subscripts e, c, h, k, and reg refer to the expansion and the compression spaces, heat exchangers - heater and cooler - and regenerator.

A supplementary hypothesis is needed for the compressor: the valves are opened and closed instantaneously, at null pressure differences.

The opening of the suction valve takes place when inside the cylinder the pressure drops down to the pressure inside the suction line. The suction valve closure occurs when the pressure inside the cylinder tends to increase - at the moment when the mass of gas reaches its maximum value.

The opening of the discharge valve has already taken place when the pressure inside the working space increases up to the pressure inside the discharge line. The closing of the discharge valve occurs when the pressure inside the cylinder tends to decrease - at the moment when the mass of gas reaches its minimum value. The moments of opening and closing the valves are characterized by angular position of the crankshaft αSV_o , αSV_c , αDV_o , αDV_c .

Having made the notations on Fig. 1 *b*, the gas masses inside the functional spaces are determined with the aid of the state equation:

$$m = \frac{pV}{RT}. \quad (1)$$

Summing up the relations given by the state equation and assigning m_T to the total mass of agent, the momentary pressure relation is obtained as

$$p(\alpha, m_T) = \frac{m_T R}{\frac{V_c(\alpha) + V_k}{T_k} + \frac{V_{reg}}{T_{reg}} + \frac{V_e(\alpha) + V_h}{T_h}}, \quad (2)$$

where R is the working fluid constant, the subscript T means total and α is the crankshaft rotation angle.

According to the isothermal physico-mathematical model of the EDSC, the pressure inside the cylinder is given by the following function

$$p(\alpha) = \begin{cases} p_S & \text{if } \alpha \in (\alpha_{SV_o}, \alpha_{SV_c}) \\ p_c = p(\alpha, m_{\max}) & \text{if } \alpha \in (\alpha_{SV_c}, \alpha_{DV_o}) \\ p_D & \text{if } \alpha \in (\alpha_{DV_o}, \alpha_{DV_c}) \\ p_e = p(\alpha, m_{\min}) & \text{if } \alpha \in (\alpha_{DV_c}, \alpha_{SV_o}). \end{cases} \quad (3)$$

During the compression process the working space contains the maximum mass m_{\max} of the entire cycle, and the pressure is p_c . During the expansion process, inside the cycle the minimum mass m_{\min} is evolving, and the pressure is p_e . The mass difference ($m_{\max} - m_{\min}$) enters inside the working space during suction and leaves the cycle during the discharge process.

Using eq. 3 it is possible to calculate the indicator diagrams $p(V(\alpha))$, $p(V_c(\alpha))$ and $p(V_e(\alpha))$, for which notation $V(\alpha)$ stands for the total volume of the engine.

The work yielded per cycle is calculated with:

$$L = \oint p(V(\alpha)) d(V(\alpha)) = \int_0^{2\pi} p(V(\alpha)) \left[\frac{d}{d\alpha}(V(\alpha)) \right] d\alpha. \quad (4)$$

Relations similar to eq. 4 are used to obtain the work done by the expansion space L_e and the work done by the compression space L_c .

The efficiency of the Stirling engine can be obtained with the following relation:

$$\eta_S = 1 - \frac{|L_c|}{L_e}. \quad (5)$$

Numerical example. A beta-type engine-driven Stirling compressor is considered (Fig. 1). The EDSC is characterized by the following main dimensions: crank radius $r = 0.0365$ m, connecting rod length $l = 0.15$ m, gear offset $e = 0.046$ m, distance between the lower yoke and the frontal surface of the displacer $l_{sd} = 0.8135$ m, distance between upper yoke and the frontal surface of the displacer $l_{sp} = 0.363$ m, displacer and power piston diameter $D = 0.073$ m, displacer stem diameter $d = 0.02$ m, displacer length 0.15 m, distance between the displaced in TDC (top dead center) and the cylinder head $f = 0.002$ m, ratio between the volume of the cooler and the volume swept by the displacer 0.1 , ratio between the volume of the regenerator and the volume swept by the displacer 1.2 , ratio between the volume of the heater and the volume swept by the displacer 0.1 .

The functional parameters of the simulated working regime are: heater temperature $T_h = 773$ K, cooler temperature $T_k = 310$ K, logarithmic mean temperature of the regenerator $T_{reg} = 506.7$ K, revolution speed $n = 1000$ rpm.

The working agent is air, and the suction pressure is the atmospheric pressure, $p_S = p_{atm} = 101325$ Pa.

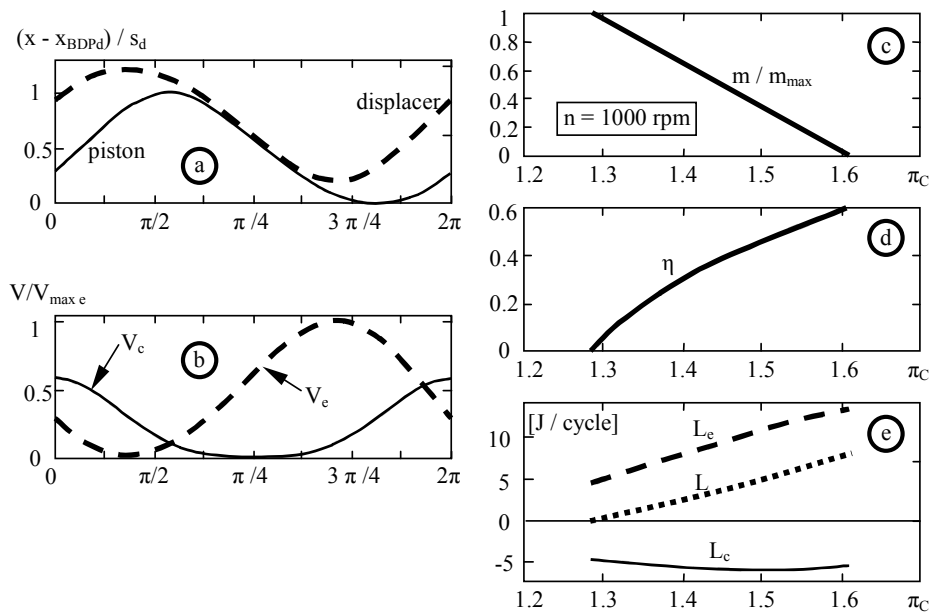


Fig. 2 – Cyclic variation of the functional performances of beta-type EDSC: *a* - relative position of the pistons; *b* - dimensionless variation of the compression and expansion chambers volumes; *c* - dimensionless variation of the discharged mass flow as function of the compression ratio $\pi_c = p_D/p_S$; *d* - engine efficiency as function of the compression ratio; *e* - variation of work exchanged as function of compression ratio.

The maximum air mass inside the engine-driven compressor (at the end of suction process) is 0.6 g. The minimum mass (at the end of the discharge process), calculated for the null output work and for the minimum discharge pressure, is 0.3725 g.

In Figs. 2 and 3 the results calculated with the isothermal model applied to the EDSC with the previously mentioned geometric and functional parameters are summarized.

The pure compressor operating mode, Fig. 3 *a*, is obtained when the work cyclically spent inside the compression chamber is equal with the work yielded cyclically inside the expansion chamber. In this case the work available at the shaft of the engine is zero, while the engine efficiency is zero. In this operating mode the discharge pressure takes its minimum value, p_{Dmin} , and the discharged mass flow is the maximum one.

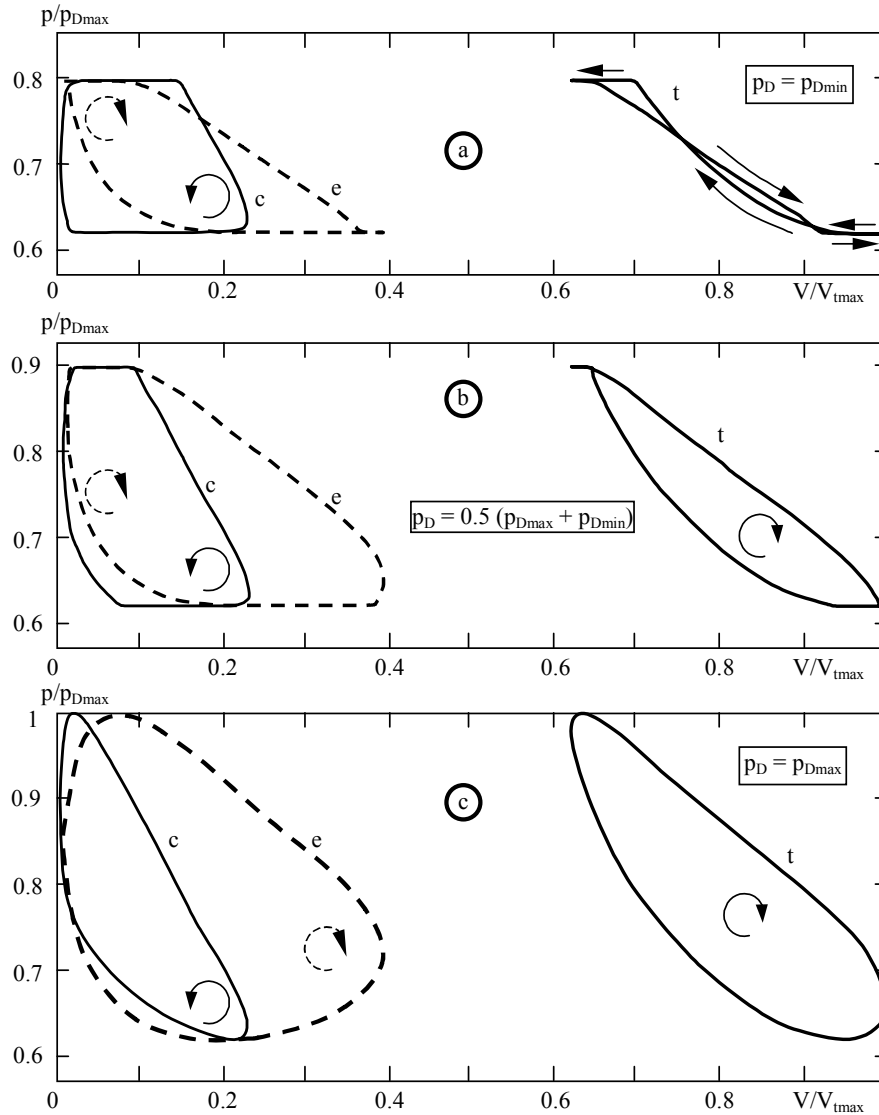


Fig. 3 – Indicator diagrams of the EDSC for different discharge pressures:
a - pure compressor; *b* - engine-driven compressor, with
 $p_D = 0.5 (p_{Dmax} + p_{Dmin})$; *c* - pure engine.

For discharge pressures $p_D < p_{Dmin}$ the EDSC cannot function, because it will need an additional external work for completing the compression process.

The pure engine operating mode, Fig. 3 *c*, is obtained when the discharge valve never opens, $p_D \geq p_{Dmax}$. The entire work yielded is available at the engine shaft, and the discharged mass flow is zero.

For discharging pressures between maximum and minimum the EDSC discharges a mass flow of compressed gas and also yields work available at the engine shaft, Fig. 3 *b*.

3. Physico-Mathematical Model of the Theoretical Stirling Engine-Driven Compressor

In order to appreciate the maximum performances of the EDSC the theoretical EDSC must be defined, Fig. 4 *a*; this machine allows us to compare different EDSC, with different geometries and different operating conditions. The introduction of the theoretical EDSC can be done in a similar way as the definition of the theoretical Stirling engine (Homutescu & Bălănescu, 2006).

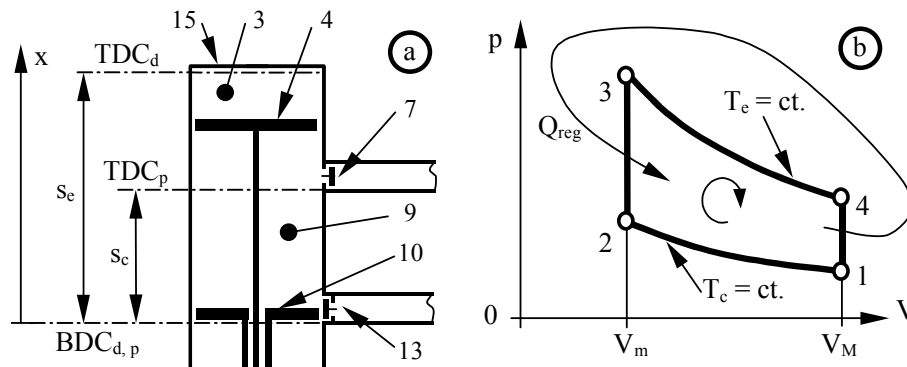


Fig. 4 – Scheme of the theoretical EDSC and the thermodynamic Stirling cycle:
3 - expansion chamber; 4 - displacer; 7 - discharge valve; 9 - compression chamber;
10 - power piston; 13 - suction valve; 15 - cylinder.

The theoretical Stirling machine is a machine in which the working agent performs the thermodynamic Stirling cycle (Fig. 4 *b*). The theoretical Stirling engine must have only two chambers (Homutescu & Bălănescu, 2006), one for compression and one for expansion. Inside the theoretical EDSC a variable mass of ideal gas evolves, but the isothermal compression and expansion processes take place with constant masses of gas. The pressure losses cannot occur and the instantaneous pressure is the same inside both machine chambers. Inside the theoretical Stirling machine the heat transfer is realized through portions of the cylinder and of the cylinder head. The regenerator is represented also by a portion of the cylinder walls.

The duration of the functional phases are arbitrary (Homutescu & Bălănescu, 2006), because at the theoretical machine the performances are given by the law of variation of pressure as function of volume and not of time. The piston movement laws are also arbitrary, for the same reason: the performances depend only on the $p(V)$ variation law. The periods during which

the pistons remain stationary and the periods during which the pistons have correlated movements depend on the constructive scheme (Fig. 4).

The theoretical EDSC attached to the beta-type EDSC with rhombic drive analyzed into the previous chapter retain the value of the cylinder diameter and the same strokes “s” of the power piston and of the displacer, the same temperatures of the heat sources, the same working agent and the same rotation speed.

The hypotheses of the physico-mathematical model of the theoretical EDSC are derived from the ones presented at the isothermal model of the beta-type EDSC. The valves are opened and closed also instantaneously, also at pressure differences equal to zero.

The discharge process begins with the opening of the discharge valve, event that takes place when the discharge pressure is obtained inside the cylinder. The discharge pressure must be achieved during the process of displacing the agent from compression to expansion chamber, Fig. 5. After the opening of the discharge valve a fraction of the working agent will leave the working space, at $p_D = \text{const}$. Until the end of the displacing process the total volume occupied by the working agent is constant, the pressure is also constant, so the discharging process will appear as a dot on p - V diagram. The closing of the discharge valve will occur at the beginning of the expansion process. During the discharging the gas mass trapped inside the EDSC diminishes from its maximum value (from the end of the suction process and during the compression) to its minimum value (from the end of discharging and during the expansion).

The expansion process is identical with the expansion inside the regular Stirling engine. It is a process that takes place with constant mass. The whole mass of gas (minimum for the entire cycle) is trapped inside the expansion chamber.

During the process of displacing the gas from expansion to compression chamber the pressure decreases. The suction valve opens when the pressure inside the working space decreases to the suction pressure p_s , according to the adopted hypotheses. From this moment on the pressure remains constant and a mass of fresh gas enters into the machine. At the end of the displacing process the entire mass of gas will be inside the compression chamber (the maximum gas mass during the cycle). At the beginning of the compression process the suction valve closes. The suction process takes place at constant pressure and at constant total volume, so the suction will be represented as a dot into the p - V diagram.

The compression process implies the decrease of the volume of a constant mass (maximum for the entire cycle) of gas, similarly to the reciprocating compressors.

During the displacement of the working agent from compression to expansion chamber the pressure rises, due to the heat received in a process that takes place at constant total volume.

Therefore, the process of increasing the pressure inside the EDSC takes place in two phases:

- through mechanical compression, during process 1-2, when the entire mass of agent is inside the compression chamber
- through heating of the working agent, during the displacement from the compression to expansion chamber (process 2-3).

The variation of the instantaneous pressure inside the theoretical machine is obtained from the eq. 1 of the conservation of the mass, and is defined on several intervals:

$$p(\alpha) = \begin{cases} \frac{m_{max} RT_k}{V_c} & \text{if } \tau \in \left(0, \frac{\Theta}{4}\right) \\ \frac{m_{max} R}{\frac{V_c}{T_k} + \frac{V_e}{T_h}} & \text{if } \tau \in \left(\frac{\Theta}{4}, \tau_{DVo}\right) \\ p_D & \text{if } \tau \in \left(\alpha_{DVo}, \frac{2\Theta}{4}\right) \\ \frac{m_{min} RT_h}{V_e} & \text{if } \tau \in \left(\frac{2\Theta}{4}, \frac{3\Theta}{4}\right) \\ \frac{m_{min} R}{\frac{V_c}{T_k} + \frac{V_e}{T_h}} & \text{if } \tau \in \left(\frac{3\Theta}{4}, \tau_{SVo}\right) \\ p_S & \text{if } \tau \in (\alpha_{SVo}, \Theta) \end{cases} \quad (6)$$

where τ is time and Θ – the period.

The works exchanged inside the chambers and inside the machine are obtained with the defining relation:

$$L = \oint p dV. \quad (7)$$

From the first law of the thermodynamics we obtain that, for either the compression or expansion chamber, during one complete cycle, the heat exchanged is equal to the work exchanged.

The efficiency of the engine can be calculated with the eq. 5. The efficiency is equal with the efficiency of the Carnot cycle evolving between the same temperatures (due to the ideal heat regeneration) for the pure engine only.

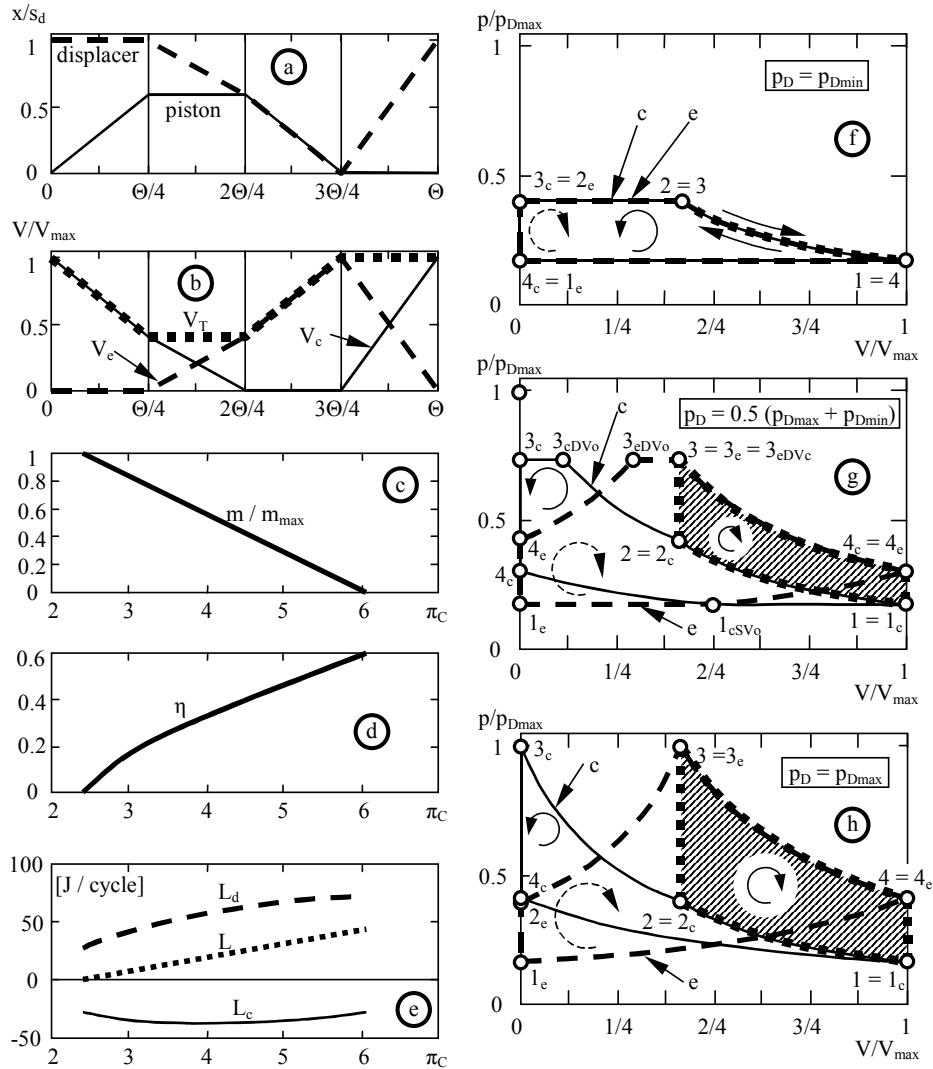


Fig. 5 – Cyclic variation of the functional performances of theoretical beta-type EDSC: *a* - relative position of the pistons; *b* - dimensionless variation of the compression and expansion chambers volumes; *c* - dimensionless variation of the discharged mass flow as function of the compression ratio $\pi_c = p_D/p_S$; *d* - engine efficiency as function of the compression ratio; *e* - variation of the works exchanged as function of the compression ratio; *f* - indicator diagrams for pure compressor; *g* - indicator diagrams for engine-driven compressor, with $p_D = 0.5 (p_{Dmax} + p_{Dmin})$; *h* - indicator diagrams for pure engine.

Numerical example. The beta-type theoretical engine-driven Stirling compressor derived from and associated to the EDSC presented above is characterized by the following main dimensions: displacer and power piston

diameter $D = 0.073$ m, displacer stroke 0.07698 m and power piston stroke 0.04519 m.

The functional parameters of the simulated working regime are the same: $T_h = 773$ K, $T_k = 310$ K, $T_{reg} = 506.7$ K, $n = 1000$ rpm. The working agent is also air, and the suction pressure is $p_s = 101325$ Pa.

The maximum air mass inside the engine-driven compressor (at the end of suction process) is 0.367 g. The minimum mass (at the end of the discharge process), calculated for the null output work and for the minimum discharge pressure, is 0.147 g.

In Fig. 5 the results calculated for the theoretical EDSC with the previously mentioned geometric and functional parameters are summarized. The indicator diagrams were drawn for discharge pressures similar to the ones previously used.

Fig. 5 g is the most relevant one for the theoretical EDSC, because it clearly shows the suction and discharge processes inside compression and expansion chambers - processes that appear as line segments, of constant pressure.

4. Conclusions

1. A scheme for achieving an engine-driven Stirling compressor has been shown, by introducing automatic valves located in the zone of the cooler heat exchanger. Placing the discharge valve in the zone of the cooler implies that the temperature of the discharged gas is reduced, and eliminates the necessity of a final cooler heat exchanger for the compressor.

2. By applying an isothermal physical-mathematical model, it was found that there is only one operating regime for which the EDSC works only as compressor. This unique regime is characterized by the minimum discharge pressure (by the minimum compression ratio) and by the maximum mass flow that the EDSC can deliver. For pressures lower than p_{Dmin} the engine cannot yield, through expansion, sufficient work for covering the work spent for the compression. For pressures larger than p_{Dmin} and lower than the maximum discharge pressure the EDSC delivers a mass flow of gas and also yields work to the crankshaft, simultaneously. For the maximum discharge pressure the discharge valve never opens (similarly to the regular reciprocating compressors). At this operating regime the machine functions as a pure Stirling engine.

3. The increasing of the pressure inside the EDSC is obtained through the decreasing of the volume and also through the heating of the fraction of the working agent displaced inside the expansion chamber (from the lower temperature, to higher temperatures).

4. Due to the defining conditions - annulling the external heat exchanger volumes, theoretical movements of pistons etc. - the performances of the theoretical EDSC are the maximum ones.

5. Both physico-mathematical models proposed allow for establishing the maximum performances that can be achieved by this unusual Stirling machine.

REFERENCES

- Homutescu C.-A., Savitescu Gh., Jugureanu E., Homutescu V.M., *Introducere în mașini Stirling*. Ed. CERMI, Iași (2003).
- Homutescu V.M., Bălănescu D.-T., *Theoretical Realization of the Stirling Cycle on Two Pistons Machines*. The 2nd International Conference “Advanced Concepts in Mechanical Engineering”, June 2006, Bul. Inst. Polit. Iași, **LII (LVI)**, 6C, 143–150 (2006).
- Homutescu V.M., Homutescu C.-A., Homutescu A., *Appreciations About a Variable Displacement Stirling Engine*. Termotehnica, V, 2/2001, 58–62 (2001).
- Popescu Gh., *Mașini Stirling*. Ed. Bren, București (2001).

PERFORMANȚELE MAXIME ALE MAȘINII STIRLING FUNCȚIONÂND ÎN REGIM DE MOTOCOMPRESOR

(Rezumat)

Este analizată, pe baza unor modele fizico-matematice izotermice, funcționarea unui motocompresor Stirling. Sunt determinate performanțele ce pot fi atinse de mașina Stirling teoretică (ce funcționează după ciclul termodinamic Stirling) și performanțele unei mașini construite după schema beta. Aceste performanțe au caracter de performanțe maxime și pot fi folosite ca referință pentru compararea mașinilor reale între ele.